

Matrices

Practice Questions – Worked Answers

Instructions

The following answers show the necessary working for each question.

1. Given the matrices

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix},$$

- (a) To add two matrices, add corresponding entries:

$$\begin{aligned} A + B &= \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 2+5 & 3+1 \\ 1+2 & 4+3 \end{pmatrix} = \boxed{\begin{pmatrix} 7 & 4 \\ 3 & 7 \end{pmatrix}} \end{aligned}$$

- (b) Multiply every entry of A by 2:

$$2A = 2 \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \boxed{\begin{pmatrix} 4 & 6 \\ 2 & 8 \end{pmatrix}}$$

- (c) For matrix multiplication, each entry is found by multiplying a row of A by a column of B .

$$\begin{aligned} AB &= \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix} \\ &= \begin{pmatrix} (2)(5) + (3)(2) & (2)(1) + (3)(3) \\ (1)(5) + (4)(2) & (1)(1) + (4)(3) \end{pmatrix} \\ &= \begin{pmatrix} 16 & 11 \\ 13 & 13 \end{pmatrix} \end{aligned}$$

Therefore,

$$AB = \begin{pmatrix} 16 & 11 \\ 13 & 13 \end{pmatrix}$$

2. (a) The quantities matrix is

$$Q = \begin{pmatrix} 150 & 100 & 0 & 10 \\ 100 & 120 & 50 & 20 \end{pmatrix}$$

The rows represent the two meals and the columns represent chicken, rice, potato and olive oil respectively.

- (b) The nutritional information matrix is

$$N = \begin{pmatrix} 1.65 & 0.31 \\ 1.30 & 0.027 \\ 0.35 & 0.024 \\ 8.84 & 0 \end{pmatrix}$$

The first column gives calories per gram and the second column gives protein per gram.

- (c) We calculate

$$QN = \begin{pmatrix} 150 & 100 & 0 & 10 \\ 100 & 120 & 50 & 20 \end{pmatrix} \begin{pmatrix} 1.65 & 0.31 \\ 1.30 & 0.027 \\ 0.35 & 0.024 \\ 8.84 & 0 \end{pmatrix}$$

For the first meal:

$$150(1.65) + 100(1.30) + 0(0.35) + 10(8.84) = 465.9$$

and

$$150(0.31) + 100(0.027) + 0(0.024) + 10(0) = 49.2.$$

For the second meal:

$$100(1.65) + 120(1.30) + 50(0.35) + 20(8.84) = 515.3$$

and

$$100(0.31) + 120(0.027) + 50(0.024) + 20(0) = 35.44.$$

Therefore,

$$QN = \begin{pmatrix} 465.9 & 49.2 \\ 515.3 & 35.44 \end{pmatrix}$$

- (d) The first row represents the Chicken and Rice meal:

$$465.9 \text{ kcal and } 49.2 \text{ g of protein}$$

The second row represents the Chicken Biryani:

$$515.3 \text{ kcal and } 35.44 \text{ g of protein}$$

3. (a) The average view percentages can be written as the matrix

$$R = \begin{pmatrix} 0.42 & 0.53 & 0.87 \\ 0.56 & 0.44 & 0.71 \\ 0.46 & 0.61 & 0.69 \\ 0.39 & 0.43 & 0.47 \end{pmatrix}$$

Each row represents a video and each column represents an audience group.

- (b) There are three audience groups, so their proportions can be represented by a 3×1 column matrix:

$$P = \begin{pmatrix} 0.20 \\ 0.25 \\ 0.55 \end{pmatrix}$$

The dimensions are compatible for matrix multiplication because

$$(4 \times 3)(3 \times 1) = (4 \times 1).$$

Also,

$$0.20 + 0.25 + 0.55 = 1,$$

so the three proportions account for the entire audience.

(c) We calculate

$$\begin{aligned} RP &= \begin{pmatrix} 0.42 & 0.53 & 0.87 \\ 0.56 & 0.44 & 0.71 \\ 0.46 & 0.61 & 0.69 \\ 0.39 & 0.43 & 0.47 \end{pmatrix} \begin{pmatrix} 0.20 \\ 0.25 \\ 0.55 \end{pmatrix} \\ &= \begin{pmatrix} (0.42)(0.20) + (0.53)(0.25) + (0.87)(0.55) \\ (0.56)(0.20) + (0.44)(0.25) + (0.71)(0.55) \\ (0.46)(0.20) + (0.61)(0.25) + (0.69)(0.55) \\ (0.39)(0.20) + (0.43)(0.25) + (0.47)(0.55) \end{pmatrix} \\ &= \begin{pmatrix} 0.695 \\ 0.6125 \\ 0.624 \\ 0.444 \end{pmatrix} \end{aligned}$$

Therefore,

$$RP = \begin{pmatrix} 0.695 \\ 0.6125 \\ 0.624 \\ 0.444 \end{pmatrix}$$

(d) The four entries represent the weighted average view percentage for each video, taking into account the proportions of the three audience groups.

Therefore:

Video	Weighted average view percentage
The 7 Levels of Pure Mathematics	0.695
The 7 Levels of Math Symbols	0.6125
How does 2TB fit in a microSD	0.624
Ranking Every University Maths Course	0.444

Converting these to percentages:

$$0.695 = 69.5\%$$

$$0.6125 = 61.25\%$$

$$0.624 = 62.4\%$$

$$0.444 = 44.4\%$$

(e) The highest average view percentage is

$$0.695 = 69.5\%$$

Therefore,

The 7 Levels of Pure Mathematics

has the highest weighted average view percentage.